DAVID TEMPERLEY

The Line of Fifths

1. Introduction

The concept of pitch-class depends, as is well-known, on the assumption of octave equivalence: any two notes one or more octaves apart are members of the same category. But the pitch-class idea also entails another kind of equivalence, that is, enharmonic equivalence: any two notes of the same *pitch* belong to the same category. The assumptions of octave and enharmonic equivalence have proved to be extremely useful to music theory. First proposed with serial music in mind, the pitch-class system has more recently opened up a new approach to the study of tonal music as well.[1] However, we should remember that enharmonic equivalence is not the usual assumption in traditional tonal theory and music notation. There, the same pitch-class may be represented in different ways: as A♭ or G♯, for example. Retaining the assumption of octave equivalence, we might call such categories 'tonal pitch-classes' (TPCs) as opposed to the twelve 'neutral pitch-classes' (NPCs) of atonal set theory. In this article, I shall explore the tonal pitch-class system and the role of tonal pitch-class distinctions in musical cognition. In emphasising the differentness of A♭ and G♯, I am not in any way denying that they are, in another important sense, the same – any more than distinguishing between C3 and C4 denies the essential similarity between them. It seems clear that both the neutral pitch-class and tonal pitch-class systems play important roles in tonal music. However, the tonal pitch-class system has not received the theoretical attention it deserves.

I begin by proposing a very simple spatial representation of tonal pitch-classes. I then present a model of how TPC labels are inferred from neutral ones: that is, the principles for choosing one spelling of a pitch over another. I also discuss the issue of ambiguity: cases where a single event seems to be functioning as two different TPCs. In the second half of the article, I explore some of the ways in which spelling choices are consequential for other aspects of music cognition; I also consider their relevance to the problem of key-finding. In closing, I discuss the relevance of the TPC system to highly chromatic and post-tonal music.

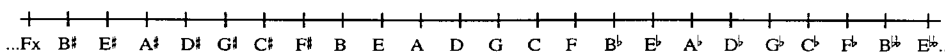
I should first mention two issues that I do *not* intend to discuss in this article. The first is intonation. The premise of this study is that inferring the spelling of notes is an important part of tonal cognition; but I will argue that this is done primarily from the *context* of the events, not their intonation. On instruments whose tuning can be spontaneously varied, different spellings of a pitch – G♯ versus A♭ – may also be *acoustically* distinguished in systematic and significant ways; and it is natural to wonder if this might affect the spellings that were perceived. However, I will not explore this possibility here. A related issue is the historical development of tonality – most notably, the evolution of temperament and tuning systems. It may seem odd to neglect this topic in view of the obvious light it sheds on the subject at hand. If – as I claim – spelling distinctions are inherent in much tonal music, and have been internalised even by modern listeners, then there can be little doubt that these distinctions were first motivated by matters of temperament: the fact that the uses of a pitch were constrained by its tuning. But this is (at the very least) another article. My concern is solely with tonal music as it is performed and heard today.

2. A Model of TPC Identification

Much work on the cognition of pitches, chords and keys has involved spatial representations. Here I shall propose a very simple representation of tonal pitch-classes – the 'line of fifths' (see Fig. 1). This is similar to the circle of fifths, except that it extends infinitely in either direction.

The line of fifths is hardly new. As early as 1821, Weber represented the circle of fifths in a spiral fashion, with A♭ and G♯ (for example) represented at the same radial angle; extending this spiral infinitely, and straightening it out, produces the line of fifths. Marx shows a fragment of the line of fifths from F♯ to G♭.[2] Other anticipations of the line of fifths are found in two-dimensional spatial representations. Riemann's 'Table of Relations' is an infinitely extending two-dimensional space, with three axes at 60-degree angles representing fifths, minor thirds and major thirds; Longuet-Higgins proposes a similar space, with two perpendicular axes for fifths and major thirds. In both of these cases, TPC distinctions are made, so that one axis of each space is essentially the line of fifths. Both Riemann's and Longuet-Higgins's models are clearly intended as models of pitch cognition; thus these are important antecedents to the current study.[3] The essential difference, of course, is the added dimension. For present purposes, the motivation for the line of fifths is that it captures distinctions – such as A♭ versus G♯ – that are cognitively important. To advocate Riemann's or Longuet-Higgins's spaces in this way, one would have to argue that distinctions even *within* the same tonal pitch-class

Fig. 1 The 'line of fifths'

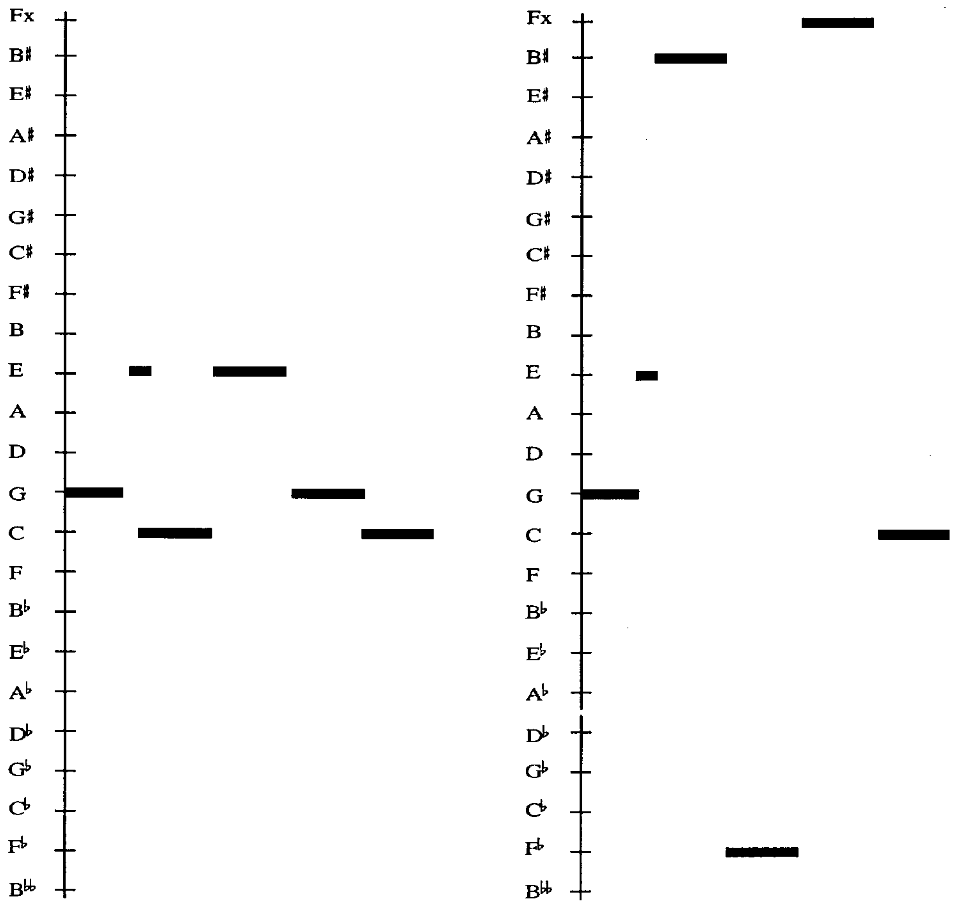


are cognitively important as well: for example, in Longuet-Higgins's space, between the A♭ four steps from C on the fifths axis, and the A♭ one step from C on the major thirds axis. (Indeed, both Riemann and Longuet-Higgins do argue along these lines.) The validity of these within-TPC distinctions is an interesting issue, but beyond the scope of this study; here I shall be concerned only with TPC categories.[4]

I propose, then, that an essential early stage of tonal cognition involves choosing TPC labels for the events of a piece, and hence mapping them on to the 'line of fifths'. The TPC of a pitch is constrained by its NPC; a pitch of NPC 0 (assuming the usual mapping) can be TPC C, B♯ or D♭♭, but not G or F♯. (This implies that the NPCs of events must also be identified.) Further criteria must be brought to bear to choose between these possibilities. In performing this process, listeners form what we might call a 'tonal pitch-class representation'. We can imagine the TPC representation of a piece as a plane, with the line of fifths on the vertical axis and time on the horizontal; each pitch-event of the piece is represented as a line segment on this plane (see Fig. 2). What follows is a proposal for a model of how these TPC representations are formed. The main evidence for such representations is simply our intuitions about how events in a piece should be spelled – for example, in writing down a melody that is dictated to us. Another source of evidence here is composers' 'orthography', that is, the way they spell notes in scores; this might be taken to indicate their TPC representations.[5] (Relying on such intuitions – our own or the composers' – is somewhat problematic, since there are cases – 'enharmonic changes' – where spelling decisions are clearly based on matters of convenience, rather than substantive musical factors. In particular, we prefer to avoid remote spellings such as triple flats and triple sharps. But I think it is usually fairly clear where such practical decisions are being made, and what the 'correct' spelling would be.) There is more indirect evidence for the reality of TPC representations as well, as I shall discuss.[6]

The model I propose is a preference rule system (similar to that of Lerdahl and Jackendoff's *Generative Theory of Tonal Music*).[7] There are three preference rules involved in choosing the preferred spelling for an event; the solution is preferred which best satisfies all the preference rules together. As input, the system requires a representation showing pitch-events in time (with their NPC labels); it produces a representation showing the TPC of each event.[8] Let us suppose the system processes a piece in a left-to-right fashion, choosing the optimal interpretation for whatever portion of the piece has been heard; with each new segment that is encountered, the system updates its analysis of everything heard so far. This naturally allows for the retrospective respelling of events; the best interpretation of an event – the one resulting in the best overall interpretation for the piece – might be affected by what happens afterwards, perhaps causing a different spelling from the one initially chosen.

Fig. 2 Two possible TPC representations of the first six pitch-events of 'The Star-Spangled Banner' (in C major).



The first rule is a very simple one, but also the most important:

> **Pitch Variance Rule.** Prefer to label nearby events so that they are close together on the line of fifths.

In many cases, this rule is sufficient to ensure correct spellings of passages. Consider the first six pitches of 'The Star-Spangled Banner'. These could be spelled G–E–C–E–G–C; alternatively, they could be spelled G–E–B♯–F♭–F𝄪–C. The first spelling is clearly preferable. The pitch variance rule offers an explanation; by this rule, the first spelling is preferred because it locates the events more closely together on the line of fifths (see Fig. 2). Note that the rule applies to nearby events. If one bar contains a B♯ followed by an E♯, there is great pressure to spell the second event as E♯ rather than F; if the two events are widely separated in time, then the pressure is much less.

While I shall not propose an absolutely precise form of the pitch variance rule (or the other rules) here, it will be useful to have some idea of how the rule can be quantified. Any TPC representation has a 'centre-of-gravity' (COG), a mean position of all the events of the piece on the line of fifths; a spelling of all the pitch-events is preferred which minimises the line-of-fifths distance between each event and the centre of gravity. In statistical terms, a representation is sought which minimises the variance among events on the line of fifths. The COG is constantly being updated; the COG at any moment is a weighted average of all previous events, with more recent events affecting it more. This ensures that the pressure for events to be located close together on the line of fifths is greatest for events that are close together in time. (One important parameter to be set here is the 'decay' value of the COG, determining how quickly the pressure to locate two events close together decays as the events get further apart in time. Roughly speaking, it seems that the pressure is significant for intervals of a few seconds, but decays quickly for longer intervals.)

The pitch variance rule is perhaps not what first comes to mind as the main principle behind spelling. Alternatively, one might explain spelling choices in terms of diatonic collections. In 'The Star-Spangled Banner', perhaps, we decide from the first few pitches that the key is C major; this implies a certain diatonic collection which then dictates the spellings of subsequent pitches. This is somewhat more complex than my explanation, since it assumes that key has already been determined and that this 'feeds back' to influence spelling. However, there are also cases where the scale-collection explanation simply does not work. Consider Ex. 1; what is the preferred spelling of the final melody note? In Ex. 1a, I hear C♯; in Ex. 1b, D♭. These choices are captured well by the pitch variance rule: C♯ is closer to the COG in the first case, D♭ in the second, though the differences are small (this can be seen intuitively in Fig. 3). To explain this in terms of scale collections, however, is problematic. The keys of the first bar in Exs. 1a and 1b are clearly C major and C minor, respectively; neither C♯ nor D♭ are in either of these scales (regardless of what 'minor scale' one chooses). One could also explain the spellings of these pitches

Ex. 1

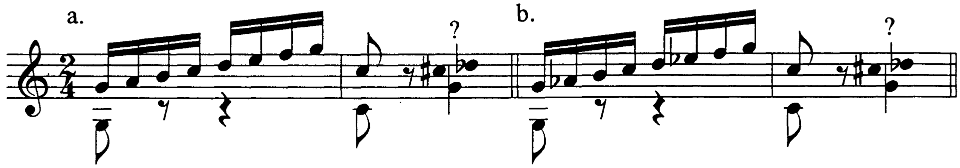
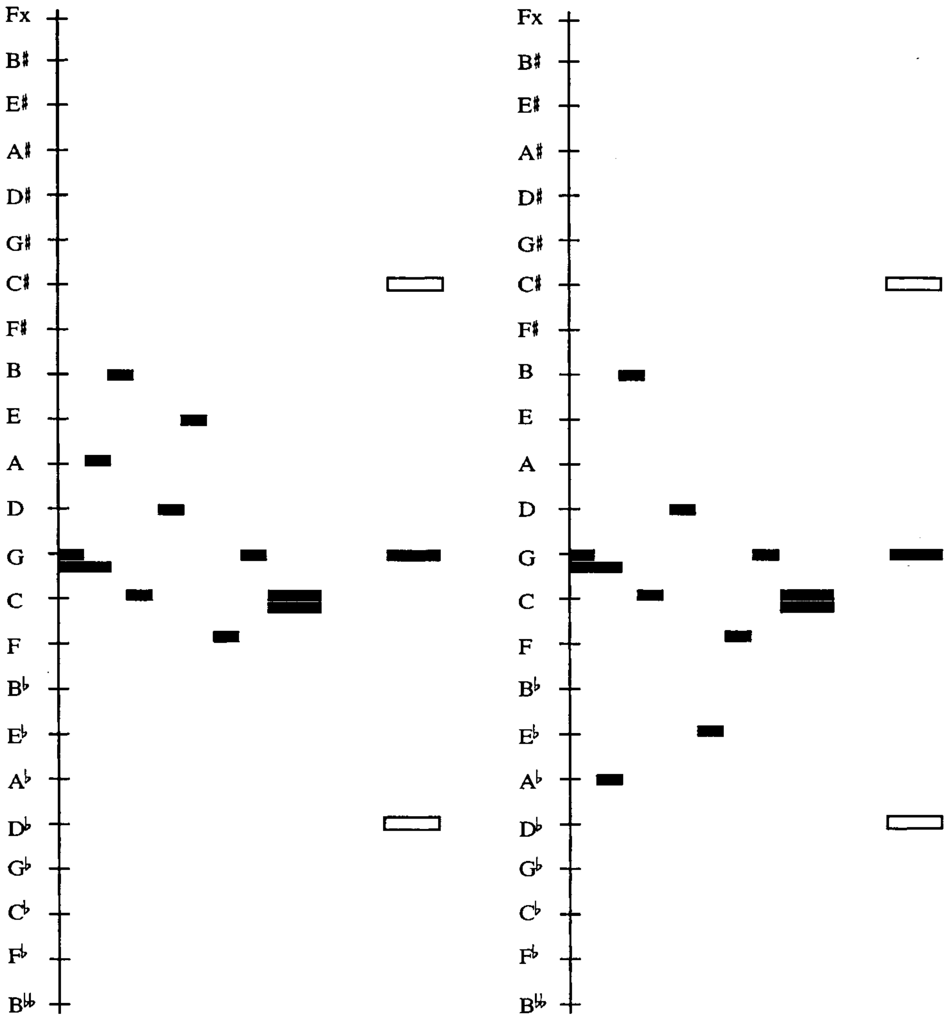


Fig. 3 TPC representations of Ex. 1. The white rectangles indicate possible spellings of the final melody note. (Line segments that are immediately adjacent indicate simultaneous events of the same TPC.)

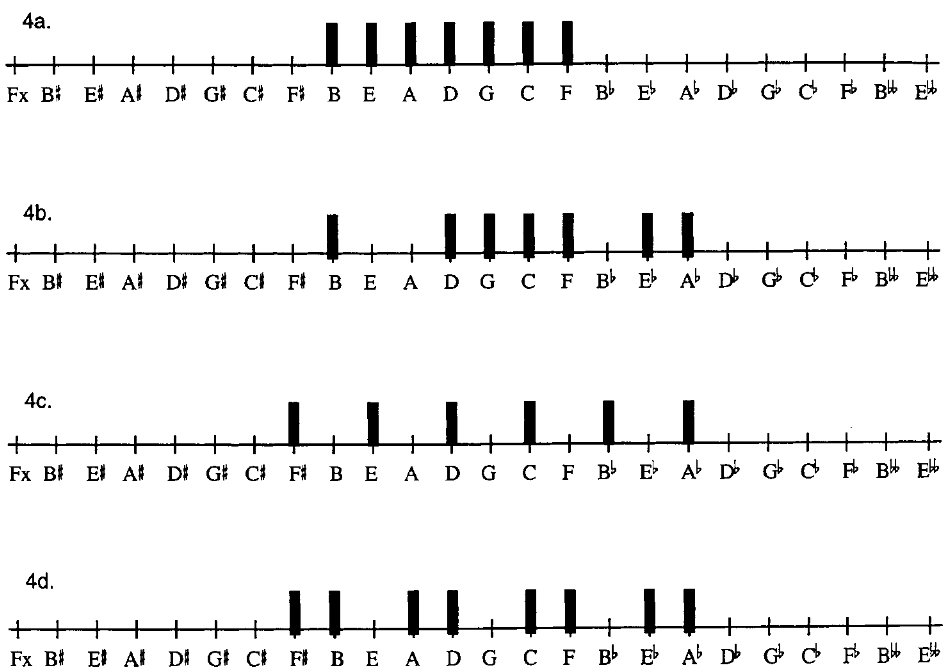


3a. (Example 1a) 3b. (Example 1b)

in terms of their harmonic and tonal implications. Clearly a C♯-G tritone implies an A7 chord, and anticipates a move to D minor; a D♭–G tritone implies E♭7, moving to A♭ major. One might claim that C♯ is preferred in the first case because the implied key of D minor is somehow closer to C major – or more expected in a C major context – than A♭ major; these key expectations then govern our interpretation of the pitches. Again, this is a rather complex explanation of something that can be explained very easily using the one rule proposed above. One of the main claims of this study is that spelling can be accomplished without relying on top-down key information. (Top-down information from harmony is sometimes required, however, as I shall discuss.)

While the pitch variance rule does not explicitly refer to diatonic collections, such collections do emerge as privileged under this rule. As is well known, a diatonic scale consists of seven adjacent positions on the circle (or line) of fifths; a passage using a diatonic scale collection will therefore receive a relatively good score from the pitch variance rule (as opposed to, for example, a harmonic minor, whole-tone or octatonic collection), since its pitches permit a very compact spelling on the line of fifths (see Fig. 4). Note also that the pitch variance rule will naturally find the *correct* spelling of a diatonic scale, since this is the most closely-packed one. For example, imagine a passage whose pitches are all (potentially) within the C major scale, with all seven steps of the scale equally represented. Assume the C major spelling of the events is chosen;

Fig. 4 Scale collections represented on the line of fifths. The representation shown for each collection is the optimal one according to the pitch variance rule. 4a – the diatonic collection; 4b – the harmonic minor collection; 4c – the whole-tone collection; 4d – the octatonic collection.



in this case, the centre of gravity for the passage – the mean position of events on the line of fifths – will be exactly at D, and all the events of the passage will lie three steps or fewer from the centre of gravity (see Fig. 4). (This also seems to be roughly correct in practice; for actual musical passages in the major, the COG is generally about two steps in the sharp direction from the tonic.)[9] Thus, this spelling of the events will be strongly preferred over other possible spellings. (A spelling of the entire passage in terms of the B♯ major scale will be equally preferred, as I shall discuss below.) If the passage contains a few 'chromatic' pitches, such as A♭/G♯ (but not enough to affect greatly the centre of gravity), the pitch variance rule may not express a strong preference as to their spelling, since two alternatives are roughly equally distant from the centre of gravity; other rules may then be decisive.

One apparent problem with the pitch variance rule should be mentioned. In 'The Star-Spangled Banner', the spelling G–E–C–E–G–C is clearly preferable to G–E–B♯–F♭–F𝄪–C; but what about F𝄪–D𝄪–B♯–D𝄪–F𝄪–B♯? Here, the pitches are as 'closely-packed' as in the original version; they are simply shifted over by twelve steps. This raises a subtle, but important, point. It is best to regard the TPC representation as relative rather than absolute: it is the relative positions of events on the line of fifths, not their absolute positions, that are important. Treating the line-of-fifths space as absolute would assume listeners with perfect (absolute) pitch. If we treat the space as relative, moreover, this means that a representation of a piece in C major is really no different from the same representation shifted over by twelve steps. This view of the TPC representation is not only cognitively plausible, but also musically satisfactory. There is no musically important reason for notating a piece in C major, for example, rather than B♯ major; it is simply a matter of notational convenience.

While the pitch variance rule alone goes a long way towards achieving good TPC representations, it is not sufficient. Another principle is voice-leading. Given a chromatic note followed by a diatonic one a semitone away, we prefer to spell the first note as being a diatonic semitone away from the second – that is, placing them on different diatonic steps – rather than a chromatic semitone.[10] In Ex. 2a, we prefer to spell the first two melody events as E♭-D rather than as D♯–D. The line of fifths provides a way of stating this principle. Note that a diatonic semitone (such as E♭–D) corresponds to a five-step interval on the line of fifths; a chromatic semitone (such as D♯–D) corresponds to a seven-step interval. The voice-leading rule can thus be stated as follows:

> **Voice-Leading Rule (first version).** Given two events that are adjacent in time and a semitone apart in pitch height, prefer to spell them as being five steps apart on the line of fifths.

Given two events of NPC 3 and 2 (adjacent in time and in the same octave), it is preferable to spell them as E♭–D rather than as D♯–D. This bears a certain

Ex. 2

a. b.

similarity to the pitch variance rule, in that it prefers to spell events so that they are close together on the line of fifths (five steps being closer than seven). However, the pitch variance rule applies generally to all nearby pitches, not merely to those adjacent and a semitone apart, and it is clear that a general rule of this kind is not sufficient. Consider Exs. 2a and 2b; apart from the E♭/D♯, these two passages contain exactly the same TPCs (the correct spelling of these events will be enforced by the pitch variance rule); and yet E♭ is preferred in one case, D♯ in the other. Thus, something approximating the voice-leading rule appears to be necessary.

A problem arises with the voice-leading rule as stated above. Consider Exs. 3a and 3b. The TPCs (excluding the G♯/A♭) are identical in both cases, thus the pitch variance rule expresses no preference as to the spelling of the G♯/A♭. But the voice-leading rule expresses no preference either. In the first case, spelling the event as G♯ will result in one 7-step gap (G–G♯) and one 5-step gap (G♯–A); spelling it as A♭ will merely reverse the gaps, creating a 5-step gap followed by a 7-step one. Either way, we have one 7-step gap, and hence one violation of the rule. Intuitively, the rule is clear; the chromatic event should be spelled so that it is 7 steps from the previous event, 5 steps from the following event. Again, rather than expressing this in terms of chromatic and diatonic pitches, we will express it in another way. A chromatic pitch is generally one that is remote from the current centre of gravity: the mean line-of-fifths

Ex. 3

a. b.

position of all the pitches in a passage. We therefore revise the voice-leading rule as follows:

> **Voice-Leading Rule (final version).** Given two events that are adjacent in time and a semitone apart in pitch height: if the first event is remote from the current centre of gravity, it should be spelled so that it is five steps away from the second.

In the case of this rule, it might seem that an appeal to scale collections would be a simpler solution. Why not say, simply, 'prefer to spell chromatic notes as five steps away from following diatonic ones'? In fact, this traditional rule does not correspond very well to musical practice. Consider the passage in Ex. 4, from Beethoven's Sonata Op. 31 No. 1/II (specifically the chromatic scales in bars 2 and 4). The traditional rule recommends we spell each of the chromatic notes as sharps, since this places them a diatonic semitone away from the following pitch. But notice that Beethoven uses B♭ rather than A♯, violating this rule; the other four chromatic degrees are spelled as the traditional rule would predict. This is, in fact, the general practice in the spelling of chromatic ornamental notes: ♭7/♯6 is generally spelled as ♭7, regardless of voice-leading context. Similarly, ♯4/♭5 is generally spelled as ♯4, even in descending lines.[11] The scale-collection approach is of no help here; B♭ is clearly chromatic in the context of C major, and it would seem arbitrary to posit a momentary move to F major in such cases. The current approach offers a solution. An ascending chromatic scale in a C major context, such as that in Ex. 4, presents a conflict for pitches such as D♯/E♭ and A♯/B♭. E♭ is closer to the centre of gravity than D♯, and is therefore preferred by the pitch variance rule; but D♯ is preferred by the voice-leading rule. (Recall that for a C major piece, the COG is generally around D.) Similarly with A♯ and B♭. In the latter case, however, perhaps pitch variance favours the flat spelling over the sharp one enough (since it is much closer to the centre of gravity) that it is preferred, overruling the voice-leading rule. In this way, the current model offers an explanation of why B♭ is preferred over A♯ in a C major context, even in ascending chromatic lines. The same logic might explain why F♯ is generally preferred over G♭ in a descending chromatic line in C major. Getting these results would depend on exactly how

Ex. 4 Beethoven, Sonata Op. 31 No. 1/II, bars 1–4.

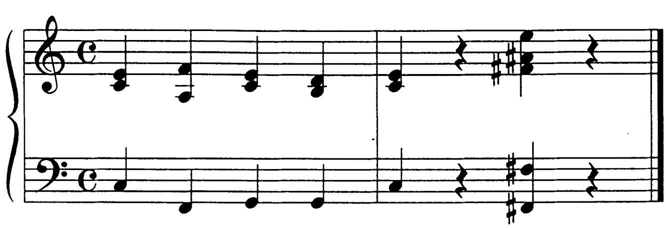
the parameters of the rules are set; it appears that, when a spelling of an event locates it within four steps of the centre of gravity, that spelling is generally preferred, regardless of voice-leading.[12]

One final rule is needed to achieve good spelling representations. Consider Ex. 5. A♯ is obviously preferred over B♭ in the final chord, but why? B♭ is clearly closer to the centre of gravity of the passage than A♯. The event in question is neither preceded nor followed by semitone motion, so voice-leading is not a factor. The explanation lies in the harmony. With A♯, the three pitches of the last chord form an F♯7 chord. As B♭, however, the pitch would be meaningless when combined with F♯ and E. (The pitches could also be spelled G♭–B♭–F♭, to form a G♭7 chord, but this is less preferred by pitch variance.) In this case, then, the TPC of the event is determined by harmonic considerations: a spelling is preferred which permits an acceptable harmony to be formed. This is enforced by the following rule:

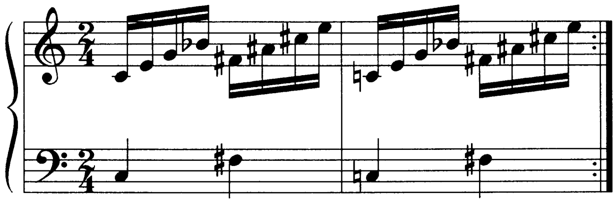
> **Harmonic Feedback Rule.** Prefer TPC representations that result in good harmonic representations.

To specify this rule in a rigorous fashion, we would obviously need to define what is meant by a 'good' harmonic representation. Elsewhere I have presented a preference rule system for harmonic analysis.[13] Under this system, a good harmonic representation is one in which a) the pitches of each segment are compatible with the chosen root; b) any non-chord notes are legal ornamental notes of some kind (such as passing notes or neighbour notes); c) roots of nearby segments are maximally close together on the line of the fifths; and d) changes of harmony occur on strong beats. The TPC representation of a piece forms part of the input to this process: whereas A♯ is considered compatible with a root of F♯, B♭ is not. (There are good reasons for assuming that TPC labels, not just NPC labels, are used for harmonic analysis, as I shall explain.) The harmonic feedback rule then says that, where other factors favour interpreting a group of events as a common tonal chord (such as a major or minor triad, or dominant or minor seventh), there will be a preference to spell the pitches accordingly; harmonic considerations thus 'feed back' to the TPC representation.

Ex. 5



Ex. 6



It may be noted that the usual 'chordal' spelling of a triad or dominant seventh is the most closely packed one on the line of fifths. Thus, one might argue that the effect of harmony on spelling is simply a consequence of the pitch variance rule: since the pitch variance rule applies most strongly to events close together in time, there is naturally very strong pressure for simultaneous events to be spelled compactly. The problem here is that the notes of a harmony are often not simultaneous. Rather, harmonic structure involves a complex process of grouping notes together (according to the principles listed above); and it is this grouping that influences how notes are spelled.

Consider Ex. 6 (imagine the pattern repeating indefinitely). The first A♯/B♭ and the second A♯/B♭ in the pattern are equally close in time to the F♯ (and also to the C, if the pattern is repeated); but the first one seems more like a B♭, the second one more like an A♯. The reason is that the metre suggests changes of harmony on the crotchet beats (rather than on the weaker beats). This groups the first A♯/B♭ together with the C–E–G, the second with the F♯–C♯–E; the harmonic feedback rule then prefers B♭ in the first case, A♯ in the second.

3. TPC Ambiguity

I have shown, in a number of cases, how the rules proposed above lead to correct spelling choices. But there is no reason to assume that there will always be one correct spelling choice for an event. Ambiguity is a well-known phenomenon in many aspects of musical cognition; it would be surprising if it never occurred in spelling. As Lerdahl and Jackendoff note, one of the advantages of preference rule systems is their ability to capture ambiguity: an ambiguous situation is one where the preference rules are, on balance, indecisive, often because some rules favour one interpretation and others favour another (individual rules may also be indecisive in themselves).[14]

There are really two kinds of TPC ambiguity to be considered. One is diachronic ambiguity: an event is first interpreted one way, but is later reinterpreted on the basis of subsequent events. Another kind is synchronic ambiguity: even after the entire context is heard, there is still uncertainty as to the correct interpretation. Diachronic TPC ambiguity – sometimes called 'enharmonic reinterpretation' – is a well-known phenomenon in music theory, acknowledged as far back as the early nineteenth century and routinely treated in theory textbooks.[15] Diachronic ambiguity frequently involves inherently ambiguous chords such as diminished sevenths and German sixths. In Ex. 7, for example, the diminished seventh on the fourth beat of the first bar is first heard as vii^{o} of G minor, with E♭ in the melody; given the following E minor chord, it is reinterpreted as vii$^{o6}_{5}$ of E minor, with D♯.

As noted earlier, preference rule systems can in principle handle diachronic ambiguity well. If we imagine the system continually updating its analysis of everything it has heard, its final interpretation of an event might differ from the one initially chosen. However, it is not clear how well the preference rule system proposed above would handle diachronic TPC ambiguities such as that in Ex. 7. In that case, it is really the voice-leading rule that should force a D♯ spelling in the last chord of bar 1. But this depends on E♭ being remote from the centre of gravity, which it is not. (The following E minor chord will cause the centre of gravity to shift; but this has no effect on the centre of gravity at the end of bar 1.) One solution would be to have the centre of gravity at each point affected by subsequent events as well as previous ones, but I shall not pursue this here.

Preference rule systems also naturally permit synchronic ambiguities, where even at a single point in time, two interpretations of an event are roughly equally preferred. To my knowledge, synchronic ambiguities of spelling have received almost no acknowledgement in music theory; the general assumption is that once the entire context is heard, the 'correct' interpretation becomes clear. (This may be partly because of our notation system, which demands a single spelling for each note.) However, I see no reason to rule out synchronic ambiguities of spelling. As the following analytical discussions will show, such ambiguities do sometimes arise, and can create situations of considerable interest.

Ex. 7

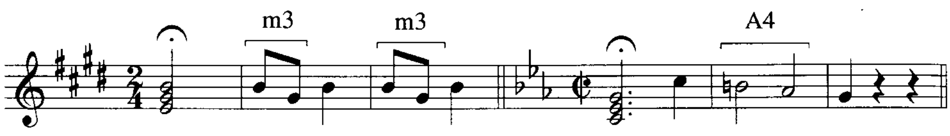
4. The Consequences of the TPC Representation

The three rules proposed earlier are largely sufficient to model our intuitive judgements about how pitch-events should be spelled. There is no doubt that composers and other trained musicians have intuitions of this kind. However, it remains to be demonstrated that these intuitions are musically important. It is possible that the TPC system is mainly a historical vestige, or, perhaps, merely a notational convenience which facilitates writing and sight-reading. I think most would agree that the role of TPC labels goes beyond this; spelling distinctions play an important role in our experience of tonal music. But what exactly is this role?

The most basic argument for the importance of TPC labels is that they are experientially real in and of themselves. In particular, the same NPC interval can have very different qualities in different TPC guises. This point is noted by Aldwell and Schachter, who offer a clear illustration (Ex. 8): the stable, placid, minor third in the first phrase has an entirely different quality from the restless, yearning, augmented second in the second phrase. This difference could be accounted for, in part, by harmonic factors. NPCs 8 and 11 can be interpreted as chord-notes of a single chord which is stable in an E major context; no such interpretation is possible in a C minor context (and note that this line of reasoning does not depend on TPC distinctions). While this may be part of the explanation, the TPC factor is surely relevant as well. In a C minor context, the preferred spelling of NPCs 8 and 11 is as A♭ and B, since this locates them maximally close to the centre of gravity (ignoring harmony and voice-leading for the moment). But in TPC terms, an interval of 9 on the line of fifths (an augmented second) is distinct from an interval of 3 (a minor third). Moreover, having two TPCs 9 steps apart, closely juxtaposed, is in itself an unstable situation by the pitch variance rule. In this way, the TPC model sheds light not only on the fact that a minor third sounds different from an augmented second, but also on the less stable quality of the latter.

Beyond the direct experiential reality of TPC labels, however, I propose that such labels have important, though often subtle, consequences for the harmonic and tonal implications of musical passages and pieces. I shall demonstrate this in several examples; first, however, two artificial passages will demonstrate the kind of phenomena that arise. In Ex. 1, I noted that the pitch

Ex. 8 From Aldwell and Schachter, *Harmony and Voice Leading*, p. 34



variance rule predicts a hearing of C♯ in the first case, D♭ in the second. But notice that this spelling choice then determines the harmonic interpretation of the final chord. With a C♯ in the melody, A7 is the preferred interpretation; with D♭, E♭7 is preferred. Such an explanation would not be possible without TPC distinctions. (It is because of such situations that it is important to allow TPC labels as input to harmonic analysis, as mentioned above.) Now let us reconsider Ex. 2. Here, the neutral pitch content of the two passages is identical. What is more, the *harmonic* content is identical as well; in both cases, the chords are clearly C major and G major. Yet the two passages have subtly different tonal implications. This is made more apparent if we consider each passage followed by two possible continuations, as shown in Ex. 9. Ex. 9c follows Ex. 9a reasonably smoothly; but Ex. 9d following Ex. 9a produces a jolt. Similarly, Ex. 9d follows Ex. 9b much more smoothly than Ex. 9c does. The voice-leading rule causes the D♯/E♭ to be heard as an E♭ in Ex. 9a, a D♯ in Ex. 9b. But this affects the tonal pitch-class content, and hence the tonal implications, of the two passages. Given the E♭, Ex. 9a is more compatible with a continuation in E♭ major, less so with E major; in Ex. 9b, the reverse is true. (I have not offered a model of how key is determined; I shall return to this issue below. For the moment, suffice it to say that one factor in key determination is the distribution of pitch-classes in a passage; however, this example suggests that it may be the distribution of tonal pitch-classes that is important for key-finding, not neutral ones.)

The role of TPC labels in tonal implication makes them a useful compositional resource. For example, a composer can endow a single neutral

Ex. 9

pitch-class with very different tonal implications by presenting it in different contexts. An example is found in Schubert's *Moment Musical*, No. 6 in A♭ major, analysed by Edward Cone.[16] Cone's analysis centres largely on the role of E/F♭. This NPC first appears in bar 12 as an E – part of a V/vi chord, suggesting a move to F minor (see Ex. 10); but instead the E moves downwards to E♭, leading to a cadence in A♭. In the middle section of the piece, this E becomes an F♭, first in several chords within A♭ minor, and then as a tonic, as the piece shifts to F♭ major (bar 29). (There is an enharmonic shift to E major here but, as Cone points out, this is a matter of 'convenience';[17] bars 29–38 are really in F♭ major, not E major. The same is true of the enharmonic shift at bar 65.) The expected return to A♭ major follows; but at the end of the piece, the F♭ again returns, leading the piece to a dark conclusion in A♭ minor. In Cone's words, the E/F♭ represents 'the injection of a strange, unsettling element into an otherwise peaceful situation',[18] an element which at first seems innocent but ultimately comes to dominate its environment.

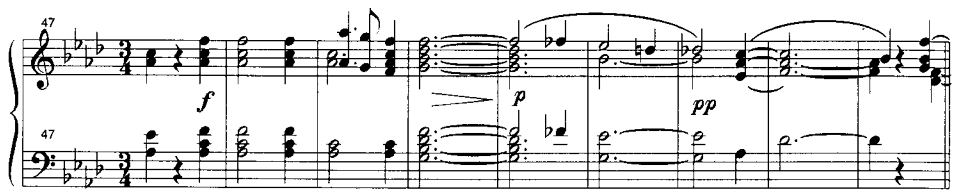
Cone's view of this piece seems entirely convincing. However, I wish to point to a few further insights that are provided by a TPC perspective. Consider the first occurrence of E/F♭ in bar 12. This melody note in bar 12 is clearly an E, as Cone says; this is ensured by the harmonic feedback rule, which favours E since it permits a C major chord. But notice that this event is

Ex. 10 Schubert, *Moment Musical*, No. 6, bars 1–20

followed stepwise by an E♭; by the voice-leading rule, this favours a spelling of F♭. While this factor is not strong enough seriously to threaten the E-ness of the note, it adds an undercurrent of tension, a subtle foreshadowing of the move in the flat direction that is to follow. (The hint of F♭ in the melody note of bar 12 is reinforced by the descending F♭–E♭ in the bass four bars later.) The F–E/F♭ gesture recurs, in the same register, at bar 51; here, however, the two spellings are in tense conflict (see Ex. 11). The harmony of bar 51 is ambivalent. It could be heard as ii^{ø}7–vii$^{o6}_{5}$ in F minor, implying an E spelling; this is surely the initial hearing, especially given the emphatic F minor chord in bars 47–9. The E interpretation is also favoured by the clear registral and metric (strong-weak) parallel with the F–E in bars 11–12. However, given what follows, the harmony in bar 51 could just as easily be heard as vii7–vii^{o}7 in A♭ major, implying an F♭ spelling (or – perhaps preferably – the F♭/E could simply be regarded as a chromatic passing note between F and E♭, again favouring F♭). Parallelism is a factor reinforcing *this* interpretation as well: hearing the downbeat of bar 51 as vii7/A♭ recalls the vii$^{4}_{3}$ at the third beat of bar 2; and in retrospect, the D in bar 52 – which is clearly a passing note – favours a passing-note hearing of the parallel F♭ in bar 51. Once again, the voice-leading rule favours the F♭ spelling, given the following E♭. On balance, I prefer the F♭ interpretation, as Schubert does; but this, it seems to me, is a real ambiguity.

Cone remarks that the F♭ in bar 51 'now seems docile, forming a passing and passive diminished seventh'.[19] Here I disagree. In the first place, it is not obvious that the F♭ is passing. If treated as an E, it is clearly parallel with the F–E in bars 11–12; seen in this way, it is not a passing note, but the resolution of an appoggiatura. But – contrary to Cone – it is precisely this non passing-note interpretation of the E that makes it 'docile'. As an E, the note implies F minor, and F minor is not a threat to A♭ major in this piece; it was dispensed with summarily in bar 13. The real conflict in this piece, so far, has been with F♭ major; and to treat this note as a passing note from F to E♭ is to treat it as an F♭, signalling the return of the sinister flat side of Cone's 'element' that is ultimately to take over the piece.

Ex. 11 Schubert, *Moment Musical*, No. 6, bars 47–55

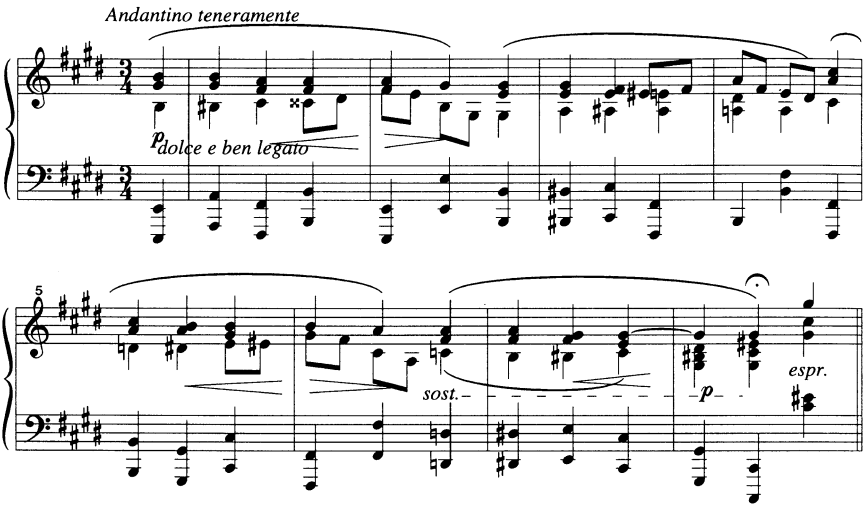


Another conflict centring on an ambiguous TPC is found in the opening of the 'Forlane' of Ravel's *Le Tombeau de Couperin*. The NPC B♯/C first appears in the left hand in bar 1, as part of an augmented triad, V + /IV; although not followed by a clear IV chord, the B♯ does resolve upwards to C♯, as expected. The B♯s in the right hand in bar 2 are more unusual. Over the A–E fifth in the left hand, there is a strong sense of an A minor chord (especially given the fact that the first chord of the piece is E minor, not major), which would imply a C hearing. In terms of voice-leading, the B♯ is followed by a B – this, too, would normally favour C over B♯. And yet Ravel's choice of B♯ here does not seem inappropriate. Perhaps the reason is that this event is heard as resolving to the C♯ a seventh below – not so much the C♯ in bar 2, but rather the one in bar 3. (The first half of bar 3 is a moment of relative repose, perhaps due to its relative harmonic clarity – the harmony here is C♯7/F♯.) The parallel with the B♯–C♯ in the left hand reinforces this hearing. (An interesting allusion to this moment occurs later on, in bar 20, where the same melodic motive is heard in the context of C minor; whereas the B♯ in bar 2 is ambiguous between ♯5 and ♭6, the A♭ in bar 20 is clearly ♭6.) The B♯/C in the second half of bar 3 (in both hands) also has an element of C, since its most likely immediate interpretation is as vii$^{o6}_{5}$ of E (the G♯ is a non chord-note, resolving to F♯ on the next beat); the resolution of this note to C♯, however, makes B♯ more plausible. Only the B♯/C in bar 4 is unambiguously a C. Again, these ambiguities of spelling have important but subtle consequences. To the extent that the B♯/C is a C, it points to the minor and to keys in the flat direction; to the extent that it is a B♯, it points in the sharp direction, suggesting the leading-note of C♯. By precariously balancing between the two, Ravel is able to gesture in both directions at once; it is partly this that gives the passage its feeling of mystery and uncertainty.

In both the Schubert and Ravel examples, a conflict arises between two spellings of the same NPC. In Brahms's Intermezzo, Op. 116 No. 6, we find a conflict of a different nature (see Ex. 13). According to the voice-leading rule, when two neighbouring events are a semitone apart, there is pressure for them to be spelled as five steps apart on the line of fifths, particularly when the first

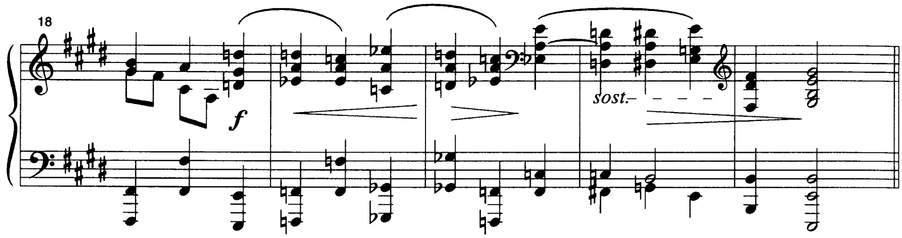
Ex. 12 Ravel, 'Forlane', from *Le Tombeau de Couperin*, bars 1–4

Ex. 13 Brahms, Intermezzo, Op. 116 No. 6, bars 1–8



event is remote from the centre of gravity. In a number of cases in the Brahms example, however, this principle is set in conflict with other factors – notably harmony – favouring another spelling. This in itself is a major source of tension in the piece. One example is the B♯/C in bar 1 (and bar 3). Voice-leading clearly prefers a B♯ spelling here; harmonically, however, the A in the bass suggests an A minor, implying a C spelling. With the C𝄪/D in bar 1 also, voice-leading favours C𝄪, while harmonic feedback favours D, since this allows a B minor seventh; in this case (more than with the B♯/C), pitch variance also strongly favours a D spelling. Although Brahms spells this as C𝄪, to my mind a D interpretation is more plausible. (It should not surprise us if there are occasional subjective disagreements as to the correct interpretation of events, as there are with every other aspect of musical structure. It seems that for Brahms, the voice-leading rule carries more weight than for me.) To the extent that this event is a D, it is ♭7 of E – a scale-degree which is highly destabilising with respect to the tonic. The issue of D versus C𝄪 returns in bar 7; in this case, the voice-leading rule (which would favour C𝄪) is clearly overridden by harmonic feedback (which permits D, since this allows a D7 chord). As ♭7, this octave D again has a disruptive effect, and suggests a move to the flat direction – which makes the following modulation in the sharp direction to C♯ major all the more surprising. The most striking conflict between harmony and voice-leading occurs in bars 19–20 (see Ex. 14). Taking the bass line in isolation, the G♭s would clearly be heard as neighbour notes to the F; however, the D harmony above the G♭ in bar 20 favours an F♯ spelling. Brahms chooses a G♭

Ex. 14 Brahms, Intermezzo, Op. 116 No. 6, bars 18–22

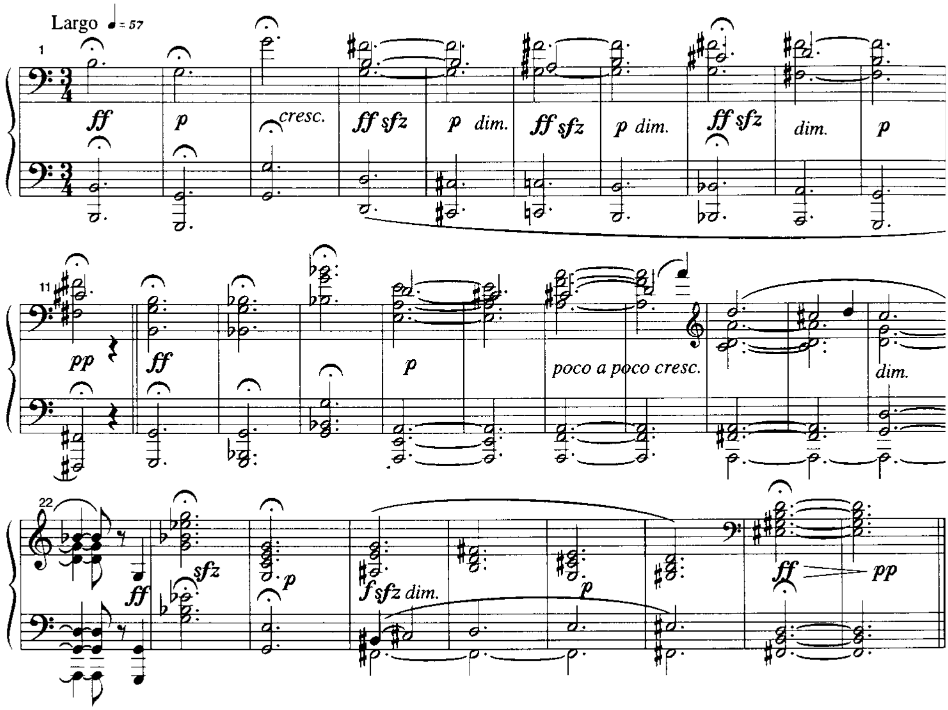


spelling here; to my mind, this note is truly ambiguous, with elements of F♯ and G♭. (The conflict is resolved on the downbeat of bar 21; here, all factors favour an F♯ spelling of the bass note.) These frequent conflicts between voice-leading and harmony give the piece an unsettling quality.

One further aspect of the TPC representation of this piece should be noted: its range on the line of fifths. The TPC furthest in the flat direction in the first two bars (if my spelling is accepted) is the D of bar 1; this is elegantly balanced by the E♯ in bar 3, far in the sharp direction, in the rhythmically parallel position to the D. The tension of bars 6–8, already discussed, is augmented by the wide TPC range of the phrase: the extremes, C and B♯, are closely juxtaposed in the alto line. (My model captures the tension caused by extreme dispersion of events on the line of fifths; this situation is low-scoring, and hence unstable, by the pitch variance rule.) In bars 19–22, the range is even greater. If we regard the F♯/G♭ in bars 19–20 as a G♭, as Brahms does (and I agree that it is at least *partly* G♭), this TPC stands 10 steps away from the tonic; with the D♯ in bar 21, the total TPC range of the passage is 16 steps, adding to the climactic intensity of the moment.

Sometimes a composer will make spelling choices that strike us, at least at first, as incorrect or counterintuitive. These choices should not be dismissed lightly, however, because they may provide clues as to how the composer thought about his or her music; in George Perle's words, they can serve as a kind of 'self-analysis'.[20] In a fascinating article, Simon Perry examines Musorgsky's sometimes unconventional orthographic practice, focusing on two movements from *Pictures at an Exhibition*.[21] For example, the fact that Musorgsky spells the opening bars of an F♯ minor piece as an A♭ major chord – when G♯ major would seem more appropriate – is seen by Perry as a way of emphasising the elusive nature of the piece's tonality. In many other cases, however, the spellings Perry discusses are perfectly logical ones, fully in keeping with conventional practice. An example is 'Sepulcrum Romanum', from *Pictures*, shown in Ex. 15. Comparing the passages in bars 12–24 and 25–30, Perry notes the contrasting spelling of several NPCs: C in bar 24 versus B♯ in bar 25; B♭ in bar 23 versus A♯ in bar 25; and F in bar 17 versus E♯ in bar 28.

Ex. 15 Musorgsky, 'Sepulcrum Romanum', from *Pictures at an Exhibition*



Perry notes that the division between the G minor spelling of the second phrase and the B minor spelling of the third seems to occur after bar 24; for this reason, he argues, we should locate the phrase boundary there, rather than after the G minor cadence in bar 22. While I fully agree about the phrase boundary, this is not a case where we need Musorgsky's actual orthography to guide us. For the tonal pitch-classes here are inherent in the music – just as they are in earlier tonal music – and are predicted well by the rules offered earlier. The F in bars 17–18 (part of a D minor chord, the C♯ being ornamental), the B♭ in bar 23 and the C in bar 24 are all dictated by harmony, as is the A♯ in bar 25; the B♯ in bar 25 is required by the voice-leading rule. As for the E♯ in bar 28, the general move to a B minor TPC collection means that E♯ is preferred over F by pitch variance (pitch variance is of course a factor in the other spellings here as well). That the phrase boundary occurs after bar 24 is quite clear from listening to the passage – and the drastic shift in TPC content at this moment (from an extreme of E♭ in bar 23 to B♯ in bar 25) is an important part of its wrenching effect.

The above discussions – they are hardly analyses, since they focus only on one limited aspect of the passages in question – reveal a number of details of structure which seem to depend crucially on tonal pitch-class distinctions. These discussions depend on the line-of-fifths model in another way, however, which is less obvious, but profoundly important. An essential part of tonal music is the sense of motion among pitches, chords and keys. It is well accepted that one aspect of this motion is movement on the circle of fifths, so that elements a fifth apart are close, and those several fifths apart are distant. Given this space of fifths, a tonal piece can take us on a complex journey, beginning at one point and making sojourns of more or less importance to other points, more or less remote, in one direction or the other; this journey may also be hierarchical, with lesser goals transitional, or subordinate, to primary ones. In the Schubert example, a fleeting move in the sharp direction – to F minor – occurs at the beginning; this is followed by a more extended journey in the other direction, to F♭ major; the return to the tonic is followed by a second move to F minor (more substantial than the first), and a final excursion in the flat direction, this time through F♭ to a slightly more distant destination, B♭♭ major.[22] Building on Cone's analysis, I described how a single pitch-class, E/F♭, plays important roles in both the sharpwards and flatwards journeys of the piece. That both E and F♭ are the same neutral pitch-class adds a fascinating extra twist, which Schubert skilfully exploits. But the more essential fact about E and F♭ is their difference: they lie in opposite directions from the tonic, and hence carry different tonal implications. The problem with using the circle as the space to represent such journeys is that, on a circle, the ideas of direction and distance have no meaning. The distance between two points, and the direction from one to the other, depends entirely on how one travels. It is tempting to say, 'but the distance between two points *is* represented on a circle: it is simply the shortest distance between them'. But, as we have seen, the distance between two points is *not* always equal to the shortest circle-of-fifths distance between them. The interval B–G♯ is different from B–A♭; a circular space has no way of representing this. But if one equates the circle distance between two points with the shortest distance between them, one is really projecting the circle on to a line, privileging one way of travelling between the two points over all the other possible ones. The circle, in itself, is powerless to represent such relationships. This is a fundamental reason why I believe a line is necessary to capture the kind of tonal journeys that take place in tonal pieces.

5. The TPC Representation as a Factor in Key-Finding

Perhaps the most counterintuitive aspect of the TPC model proposed earlier is the fact that it does not make use of scale collections and, in general, of key information. I have pointed to several reasons why scale collections do not provide an adequate solution to spelling, at least in any straightforward way. There is a further reason for seeking to derive spelling information without reliance on key. If we assume that key information is used in deriving spellings, then we are assuming that key-finding has already been accomplished. But key-finding is a highly complex process, which is not yet fully understood; and in any case, it appears to rely substantially on TPC information. This point requires some explanation. It is widely accepted that key judgements depend on the distribution of pitch-classes in a passage. One quite successful model of key-finding is Krumhansl's key-profile model.[23] In this model, the distribution of pitch-classes in a passage is calculated and compared with a key-profile, or ideal distribution, for each key; the key whose profile best matches the pitch-class distribution of the passage is the chosen key. But Ex. 9 suggests that two passages with identical NPC distributions can have different key implications; this suggests that it is TPC, rather than NPC, distribution that determines key.[24] The TPC issue is also relevant to another model of key-finding. Butler has suggested that key-finding depends not on the kind of key profile suggested by Krumhansl, but rather on certain small subsets of pitches which are unique to a particular key.[25] For example, among major keys, the pitches F–B are unique to C major and F♯ major (considering only NPCs for the moment); the set F–G–B is unique to C major. The model encounters problems when minor keys are included; if we assume the harmonic minor scale, then F–B is included not only in C major and F♯ major but C minor, F♯ minor, E♭ minor and A minor. As Butler notes, the ambiguity is reduced if we assume that spelling has been determined.[26] In TPC terms, for example, a diminished seventh such as D♯–C is unique to one key (E minor, in this case); if treated as NPCs, however, the diminished seventh is also a major sixth, common to a number of other keys. But to adopt this solution, as Butler apparently does, is to assume exactly what I have been proposing: that spelling is prior to the determination of key.[27]

Of course, it is not necessary that the influence between spelling and key determination operates in only one direction. It is also possible that there is 'feedback' from the key level to the TPC level, so that TPC labels influence key judgements but are also influenced by them – just as I have proposed with TPC labels and harmony. However, it is not clear to me that such feedback is necessary. If the current model is correct, satisfactory spelling analyses can be obtained without the use of key information and diatonic collections. I suggest, then, that spelling information is determined independently of key, and then serves as input in key determination.

6. TPC Representations in Later Music

It is instructive to consider how the proposed model deals with more chromatic music, in which the sense of tonality begins to be seriously weakened. While one might question whether TPC labels are relevant to such music, I would suggest that they can shed considerable light on our experience of it. Consider

Ex. 16, discussed by Richard Cohn; this progression is taken from Brahms's Concerto for Violin and Cello, although as Cohn points out, it is a kind of progression often used by late nineteenth-century composers such as Liszt, Wagner, and Strauss.[28]

Let us examine the behaviour of the model proposed above on this example, considering bars 270–77. In the first place, the model will attempt to spell all the pitches so that they are maximally close on the line-of-fifths. This proves to be highly problematic. Treated simply as a scale collection, the six NPCs of the passage – 0–3–4–7–8–11 – are symmetrical on the line of fifths; they permit three equally good representations, each of which has a distance of 9 steps between the two most distant points (D♯–G♯–B–E–G–C; B–E–G–C–E♭–A♭; G–C–E♭-A♭–C♭–F♭).[29] Since the pressure is greatest to locate the most temporally proximate events as close together, we might prefer the spelling shown in Fig. 5a. However, this traverses a very wide range of the line within a short period of time. Alternatively, we could keep the roots of the chords within one cycle of the line of fifths, as in Fig. 5b; this reduces the line-of-fifths range used, but entails a large leap along the line between the second and third chords. Thus, neither of these solutions is satisfactory in terms of pitch variance. As a third possibility, we could simply spell all the events of the passage within one cycle of the line, as shown in Fig. 5c. While this is maximally favoured by pitch variance, it is strongly opposed by the harmonic feedback rule, since several of the chords are then not spelled as triads; for example, the second chord is spelled A♭–B–E♭. (The behaviour of the voice-leading rule is difficult to predict in highly chromatic contexts such as these, since it depends on the location of the centre of gravity.) In short, this passage is a highly ambiguous one, offering a number of possible TPC interpretations, none of which will be particularly 'high-scoring' or strongly preferred over the others.

Several theorists have commented on the effect of such progressions.[30] Richard Cohn uses the term 'vertigo', which I find appropriate to the

Ex. 16 Brahms, Concerto for Violin and Cello, I, bars 270–77 (reduction) (after Cohn 1996, p. 15)

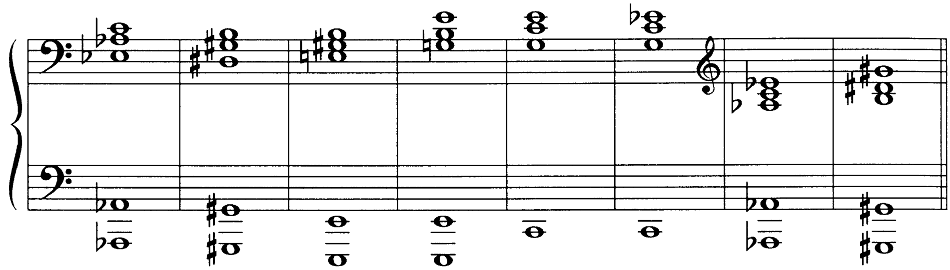
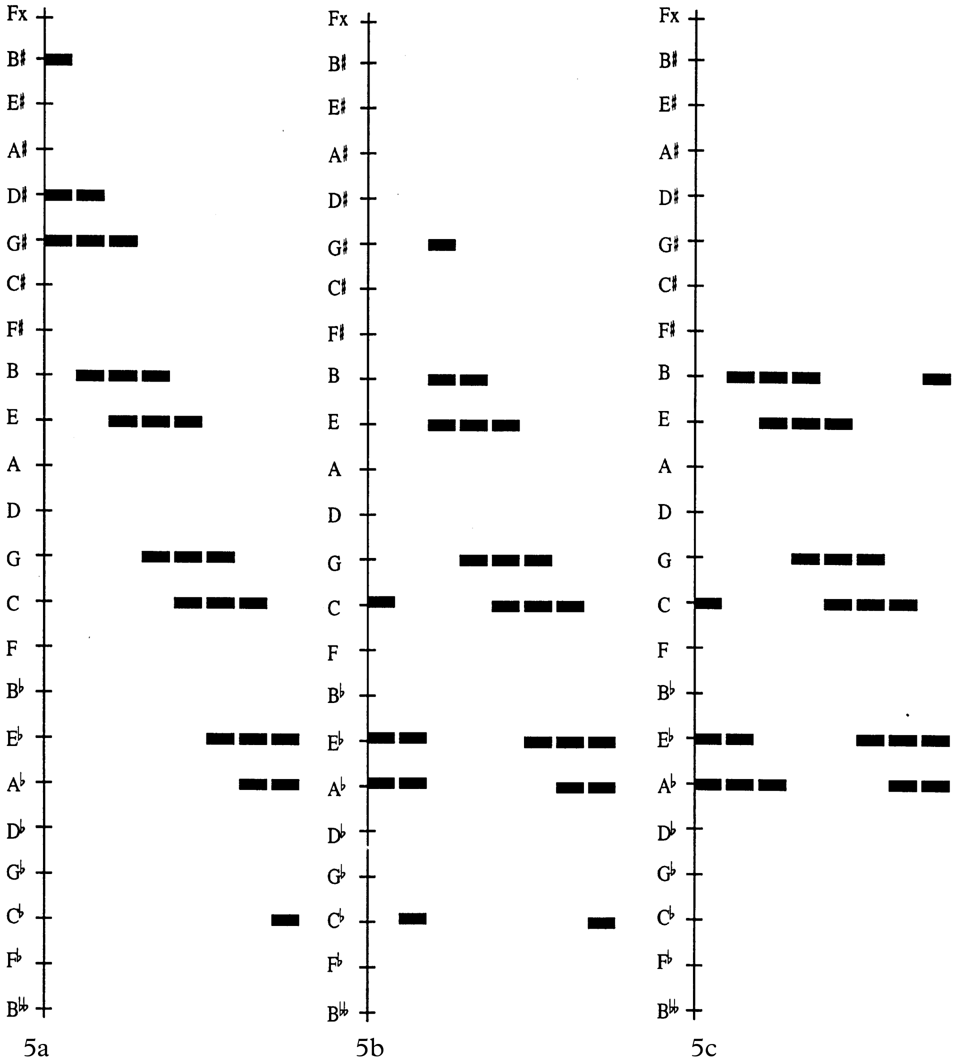


Fig. 5 Three possible TPC representations of the progression in Ex. 9 (doublings have been omitted). 5a minimises the distance between adjacent chords; 5b represents the minimum pitch variance that can be obtained while keeping each triad intact; 5c represents the minimum pitch variance overall.



experience of this passage.[31] In Cohn's view, the reason for this effect is that the progression cannot be adequately encompassed within any diatonic collection in a way that preserves the equal division of the octave.[32] My model, however, holds out a different perspective, which does not involve diatonic collections. Such passages are highly ambiguous, permitting a variety of different TPC representations; cognitively, they involve constant reinterpretation of events, searching for – but never finding – a satisfactory and clearly preferable analysis. Perhaps the most important consequence of this is that the sense of relative position of tonal elements – of distance and direction between them – is lost. It is this, I believe, that causes the sense of vertigo induced by much highly chromatic music, the sense that one does not know where one is or where one is going. This is a delicate point, because it is clearly not the case that *any* TPC ambiguity causes disorientation. We have seen – in the Schubert example, for instance – that TPC ambiguities can be successfully presented without causing complete tonal confusion. What seems to cause disorientation is situations of pervasive ambiguity, in which the relative positions of the major tonal elements of a piece or passage become unclear.

A similar point could be made about whole-tone and octatonic scales, where the same sense of disorientation has often been observed. Some theorists have explained this phenomenon in terms of what Richmond Browne has called 'intervallic context'.[33] In the diatonic scale, each position in the scale is unique with respect to its intervals with other pitches; in the whole-tone scale, all positions are the same. Thus given a particular whole-tone scale, it is impossible to distinguish different locations within the scale. Again, the current model offers an alternative (but not mutually exclusive) explanation. Like the NPC set in Cohn's example, the whole-tone scale and octatonic scales are highly ambiguous with respect to the pitch variance rule, permitting several representations that are all equally, and maximally, closely packed on the line of fifths (this is evident from the representations of these scales in Fig. 4). For example, an ascending whole-tone scale might be spelled C–D–E–F♯–A♭–B♭–C, C–D–E–F♯–G♯–A♯–B♯, C–D–F♭–G♭–A♭–B♭–C, and so on. None of the TPC preference rules expresses a strong preference between these alternatives. The disorienting effect of these scales may be due in part to their TPC ambiguity and, in particular, to the confusion of distance and direction that results from this.

I suggest, then, that the TPC model may have considerable power in explaining how we perceive highly chromatic music. It is, in part, the TPC indeterminacy of such music that gives it the quality of ambiguity and vagueness. In proposing alternative, NPC-based models for hearing late-Romantic music, Cohn seems to acknowledge that this sense of prolonged vertigo is not an entirely satisfactory way of hearing it. He points to another kind of musical logic which may have been at work in the composer's mind, a logic which was eventually to culminate in the entirely NPC-based system of serialism.[34] A similar view is reflected in the writings of other theorists – Ernst Kurth, who argues that the 'Tristan' chord, notwithstanding its complex tonal implications, also develops an enharmonic identity which transcends its various spellings; and James Baker, who finds enharmonic structures taking on increasing importance in Liszt's late works.[35] Here, too, we find the implication that late-Romantic harmony reflects a gradual shift from a harmonic (TPC) conception to an enharmonic (NPC) one – a provocative idea, for it suggests that the late-nineteenth century anticipated the twentieth not just in its abandonment of the tonal system, but in its gradual realisation of an alternative.

NOTES

1. See Richmond Browne, 'Tonal Implications of the Diatonic Set', *In Theory Only*, 5/vi–vii (1981), pp. 3–21; John Clough and Gerald Myerson, 'Variety and Multiplicity in Diatonic Systems', *Journal of Music Theory*, 29/ii (1985), pp. 249–70; Eytan Agmon, 'A Mathematical Model of the Diatonic System', *Journal of Music Theory*, 33/i (1989), pp. 1–25; Jay Rahn, 'Coordination of Interval Sizes in Seven-Tone Collections', *Journal of Music Theory*, 35/i (1991), pp. 33–60; and John Clough and Jack Douthett, 'Maximally Even Sets', *Journal of Music Theory*, 35/i (1991), pp. 93–173.

2. See Gottfried Weber, *Theory of Musical Composition*, trans. James F. Warner (London: Messrs. Cocks Co., 1851), pp. 271, 273; and Adolph Bernhard Marx, *Theory and Practice of Musical Composition*, trans. Herman S. Saroni (New York: F. J. Huntington and Mason Law, 1852), p. 47. In a way, these spaces are of limited relevance, since they apply to keys, not pitches (unlike Riemann's and Longuet–Higgins's spaces, discussed below).

   The idea of an inifinite chain of perfect fifths goes back to ancient times: see Ernest McClain, *The Pythagorean Plato: Prelude to the Song Itself* (Stony Brook, NY: N. Hays, 1978). However, unless octave equivalence is assumed, and unless positions twelve steps apart are assumed to be acoustically identical (as in an equally tempered system), such a construct has little connection to the line of fifths.

3. See Hugo Riemann, 'Ideen zu einer "Lehre von den Tonvorstellungen" ', trans. Robert W. Wason and Elizabeth West Marvin, *Journal of Music Theory*, 36/i (1992), pp. 69–117; and Christopher Longuet-Higgins, 'Two Letters to a Musical Friend', *The Music Review*, 23 (1962), pp. 244–8, 271–80. (See also Christopher Longuet-Higgins and Mark Steedman, 'On Interpreting Bach', *Machine Intelligence*, 6 (1971), pp. 221–41.) For a thorough secondary study of Riemann's Table of Relations, see Michael Kevin Mooney, 'The "Table of Relations" and Music Psychology in Hugo Riemann's Harmonic Theory' (PhD diss., Columbia University, 1996).

   While Riemann's Table of Relations is primarily cognitive, it sometimes has acoustic implications as well. In Riemann's first presentation of the table, just temperament was assumed, and each point on the table was associated with different frequency. Later, after Riemann embraced equal temperament, the significance of the table became entirely cognitive (see Mooney, 'The "Table of Relations" ', pp. 146–7, 161–2). Longuet-Higgins, too, justifies his space partly on acoustic grounds (pp. 244–5).

4. The line of fifths is also discussed by Eric Regener ('On Allen Forte's Theory of Chords', *Perspectives of New Music*, 13/i (1974), pp. 191–212), and more recently by Anthony Pople ('Editorial: On Coincidental Collections', *Music Analysis*, 15/i (1996), pp. 1–7). However, these authors make no argument for the line of fifths as a cognitive model for tonal music.

   For other work on spatial representations, see Arnold Schoenberg, *Structural Functions of Harmony* (New York: Norton, 1969), p. 20; Carol Krumhansl, *Cognitive Foundations of Musical Pitch* (New York: Oxford University Press, 1990); and Fred Lerdahl, 'Tonal Pitch Space', *Music Perception*, 5/iii (1988), pp. 315–49.

5. Composers' orthography has been the subject of some study, although this mainly concerns post-tonal music; see n. 34.

6. To my knowledge, the only other algorithm for spelling that has been proposed is that of Longuet-Higgins and Steedman (see n. 3). Their model differs substantially from mine, in that it assumes that the key of a piece or section has already been determined. The task of their algorithm is to choose the spelling of any pitches that are chromatic in context. My model has little in common with theirs, although their main principle for spelling events is somewhat similar to my voice-leading rule. For further discussion, see Temperley, 'The Perception of Harmony and Tonality: An Algorithmic Approach' (PhD diss., Columbia University, 1996), pp. 43–4.

   One might expect to find rules for spelling in theory textbooks but, in general, little is said on this subject. One exception is Aldwell and Schachter's *Harmony and Voice Leading*, which I discuss below.

7. Fred Lerdahl and Ray Jackendoff, *A Generative Theory of Tonal Music* (Cambridge, MA: MIT, 1983).

8. This model has been developed into a formal algorithm. For further details about the formalisation, see Temperley, 'The Perception of Harmony and Tonality', pp. 172–201. The algorithm has also been computationally implemented, as part of a system for performing metric and harmonic analysis. For details about the implementation, which was designed by Daniel Sleator, see David Temperley, 'An Algorithm for Harmonic Analysis', *Music Perception*, 15/i (1997), pp. 31–68, and David Temperley and Daniel Sleator, 'Modeling Meter and Harmony: A Preference Rule Approach', *Computer Music Journal*, 23/i (1999), pp. 10–27. The implementation is publicly available at the web site <www.link.cs.cmu.edu/music-analysis>. My main concern here, however, is not with the formal algorithm and the implementation, but rather with the general ideas.

9. For a small body of data on this matter, see Temperley, 'The Perception of Harmony and Tonality', pp. 229, 245.

10. This rule is found in some harmony textbooks – for example, Allen Forte, *Tonal Harmony in Concept and Practice* (New York: Holt, Rinehart and Winston, 1979), p. 10; and Edward Aldwell and Carl Schachter, *Harmony and Voice Leading* (San Diego: Harcourt Brace Jovanovich, 1989), pp. 13–14.

11. This rule – that ♭7 is generally preferred over ♯6, and ♯4 over ♭5, regardless of

voice–leading – is noted by Aldwell and Schachter (pp. 472–3); see also Simon Perry, 'Rummaging through the "Catacombs": Clues in Musorgksy's Pitch Notations', *Music Analysis*, 14/ii–iii (1995), p. 227. Other authors, however, seem to accept the traditional rule, where events in descending lines are always flattened and those in ascending lines are always sharpened; see, for example, Forte, *Tonal Harmony*, p. 4, and Walter Piston, *Harmony* (New York: Norton, 1978), p. 1. A selective examination of scores suggests that Aldwell and Schachter's rule is generally correct. In Beethoven's piano sonatas, for example, consider the ascending scales in Op. 31 No. 3/II, bars 81–2, and Op. 53/I, bar 42 (as well as the example just cited); in each of these cases, ♭7 is used, but the other chromatic degrees are all sharps. Descending chromatic scales are rarer in Beethoven's sonatas, and less consistent; consider Op. 2 No. 3/I, bar 233 (end), where ♯4 and ♯1 are used, while lowered steps are used for the other four chromatic pitches. However, in single descending chromatic passing notes, ♯4 is frequently used in passing from 5 to 4 (see, for example, Op. 49 No. 2/II, bar 7), while flattened degrees are generally used between other diatonic steps.

Beethoven's spelling – like that of other composers – is somewhat inconsistent, and I do not claim that my model would predict it perfectly. But it seems to account for the facts more adequately than the traditional rule.

12. We should also consider how the current algorithm handles the spelling of events in minor passages. It is a little unclear what the conventional wisdom is on this matter. In the major, the spelling of events within the scale of the current key is of course given by the usual spelling of that scale. In the minor, however, there is not one scale collection, but three (the natural, harmonic and ascending melodic). Aldwell and Schachter offer a rule for spelling chromatic scales in the minor that seems to describe compositional practice fairly well (p. 473). Any pitch that is in any of the three minor scales is spelled accordingly, regardless of voice-leading; that is, the spellings ♭6–6–♭7–7 are always used. In addition, 3 is usually preferred over ♭4, and ♯4 over ♭5; thus, only the spelling of ♭2/♯1 depends on voice-leading. The current model points to a possible, though imperfect, explanation for this. The scale degrees 1–2–♭3–3–4–♯4–5–♭6–6–♭7–7 form a compact region of 11 steps on the line of fifths; as with the major, it may be that the spelling of events within this range is enforced by the pitch variance rule, overriding voice-leading. I am unable to explain, however, why the range in the minor should be slightly wider than in the major (where the range of TPCs whose spelling is invariant is only nine steps).

13. Temperley, 'An Algorithm for Harmonic Analysis'.

14. Regarding preference rule systems and ambiguity, see Lerdahl and Jackendoff, pp. 39–43, and Temperley, 'An Algorithm for Harmonic Analysis', pp. 58–61.

15. Both Vogler and Weber show interest in enharmonic reinterpretation. Regarding Vogler, see Floyd K. Grave and Margaret G. Grave, *In Praise of Harmony* (Lincoln, NE: University of Nebraska Press, 1987) pp. 36–7, 39–40; regarding Weber, see 'A Particularly Remarkable Passage in a String Quartet in C by Mozart [K 465 ("Dissonance")]', trans. Ian Bent, in *Music Analysis in the Nineteenth Century*, Vol. 1, ed. Ian Bent (Cambridge: Cambridge University Press, 1994), pp. 157–83.

16. Edward Cone, 'Schubert's Promissory Note', *19th-Century Music*, 5/iii (1982), pp. 233–41.

17. *Ibid.*, p. 237.

18. *Ibid.*, p. 239.

19. *Ibid.*, p. 238.

20. George Perle, 'Scriabin's Self-Analyses', *Music Analysis*, 3/ii (1984), pp. 101–22.

21. Simon Perry, 'Rummaging through the "Catacombs"'.

22. If we assume that keys are also represented on the line of fifths, the question arises as to where minor keys are relative to major ones. Here I am assuming that both minor and major keys are located in the position of their tonics; but other solutions are also possible. For discussion of this issue, see Temperley, 'The Perception of Harmony and Tonality', pp. 202–57.

23. Krumhansl, pp. 77–110.

24. There are other weaknesses with Krumhansl's model: in particular, the fact that the same set of pitches (even the same TPCs) can be arranged in ways that have different tonal implications; see David Butler, 'Describing the Perception of Tonality in Music: A Critique of the Tonal Hierarchy Theory and a Proposal for a Theory of Intervallic Rivalry', *Music Perception*, 6/iii (1989), pp. 234–6. But I am not concerned with this here.

25. Butler, 'Describing the Perception of Tonality in Music'.

26. David Butler, 'On Pitch-Set Properties and Perceptual Attributes of the Minor Mode', in Mari Riess Jones and Susan Holleran (eds.), *Cognitive Bases of Musical Communication* (Washington, DC: American Psychological Association, 1992), pp. 161–9.

27. The same point could be made in response to Daniel Harrison ('Supplement to the Theory of Augmented Sixth Chords', *Music Theory Spectrum*, 17/ii (1995), pp. 171–4). In a discussion of augmented sixths, Harrison notes that such intervals are 'nonvalent', in that they do not belong to any diatonic set; but again this is only true if spelling is assumed.

28. Richard Cohn, 'Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions', *Music Analysis*, 15/i (1996), pp. 9–40; pp. 13–15.

29. This point should not be confused with the point made earlier about absolute versus relative representations. A passage using these six NPCs – for example, an ascending line 0–3–4–7–8–11 – would have three representations that were virtually the same in pitch variance. Yet these representations would be clearly distinct, in terms of the relative line-of-fifths positions of events within them. On the other hand, two representations of a passage that feature the same pattern of TPCs in different cycles of the line of fifths – for example, the same piece represented in C major and B♯ major – are indistinguishable in relative terms.

30. See David Lewin, 'Amfortas's Prayer to Titurel and the Role of D in *Parsifal*: The Tonal Spaces of the Drama and the Enharmonic C♭', *19th-Century Music*, 7/iii (1984), pp. 336–49; see also other discussions cited in Cohn, 'Maximally Smooth Cycles', p. 11.

31. Cohn, 'Maximally Smooth Cycles', p. 11.

32. *Ibid*.

33. Browne, 'Tonal Implications of the Diatonic Set'; see also Roger Shepard, 'Structural Representations of Musical Pitch', in Diana Deutsch (ed.), *The Psychology of Music* (New York: Academic, 1982), pp. 378–9; and John Sloboda, *The Musical Mind* (Oxford: Clarendon, 1985), pp. 235–7.

34. While some composers of non-tonal music have wholeheartedly adopted an 'NPC' approach, and thus have had no use for spelling distinctions, not all have taken this route. In some cases, composers have found new uses for traditional TPC labels: for example, using them to indicate the tonal centre of an octatonic or whole-tone collection. The spellings of Scriabin and Bartók, in particular, have been found to hold significance in this way; see Perle, 'Scriabin's Self-Analyses'; Malcolm Gillies, 'Pitch Notations and Tonality: Bartók', in Jonathan Dunsby (ed.), *Models of Musical Analysis: Early Twentieth-Century Music* (Oxford: Blackwell, 1993), pp. 42–55; and Cheong Wai-Ling, 'Orthography in Scriabin's Late Works', *Music Analysis*, 12/i (1993), pp. 47–69. While some connections could be drawn between Scriabin and Bartók's use of TPC labels and the traditional use, I would hesitate to apply the current model to their music. As noted earlier, given the inherent TPC ambiguity of octatonic collections, our perception of them is likely to be highly ambiguous in TPC terms; and even if it were not, it is not clear that the line of fifths model has much relevance to Bartók's and Scriabin's purposes. In any case, this issue is beyond my scope.

35. Ernst Kurth, *Romantische Harmonik und ihre Krise in Wagners 'Tristan'* (Hildesheim: Georg Olms Verlagsbuchhandlung, 1968), pp. 62–70; and James Baker, 'The Limits of Tonality in the Late Music of Franz Liszt', *Journal of Music Theory*, 34/ii (1990), pp. 145–73. See also Gregory Proctor, 'Technical Bases of Nineteenth-Century Tonality: A Study in Chromaticism' (PhD diss., Princeton University, 1978). Proctor argues that while a diatonic conception of tonality (which corresponds to my line-of-fifths model, although he does not describe it in these terms) is appropriate for earlier tonality, mid- and late-nineteenth century tonality requires an enharmonic conception (that is, the neutral pitch-class model).

